Modeling coherent preference relations in decision-making under risk

Arcady Novosyolov
KGTEI, SFU
arcady@novosyolov.ru

Risk measure

- Probability space $(\Omega, \mathcal{A}, \mathbf{P})$
- Random variable $X:\ \Omega \to \mathbf{R}$
- Set of random variables $\mathcal{X}$
- Risk measure $f:\ \mathcal{X} \to \mathbf{R}$

Coherent risk measures

- M $X \le Y \Rightarrow f(X) \le f(Y)$
- PH $f(\lambda X) = \lambda f(X), \quad \lambda \ge 0$
- TI $f(X + aI_\Omega) = f(X) + a, \; a \in \mathbf{R}$
- SA $f(X+Y) \ge f(X) + f(Y)$

$$I_A(\omega) = \begin{cases} 1, & \omega \in A \\ 0, & \omega \notin A \end{cases}$$

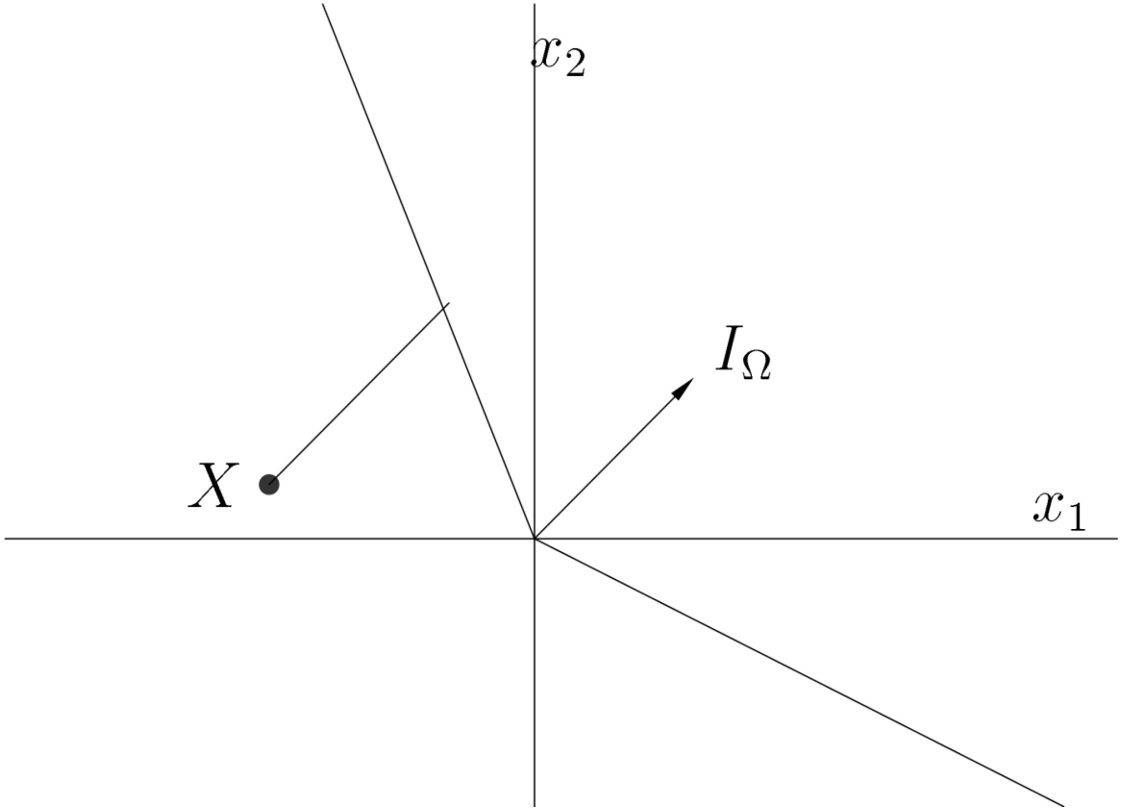
Admissible set

$$A = A_f = \{X \in \mathcal{X} : \ f(X) \geq 0\}$$

- A - convex closed cone
- $A \supseteq C_+ = \{X : \ X \geq 0\}$ positive admissible
- $A \bigcap C_{--} = \emptyset$ negative inadmissible

$$f(X) = f_A(X) = \max\{b : \ X - bI_\Omega \in A\}$$

Admissible set (cont)



Representation theorem

f is a coherent risk measure if and only if there exists a set of probability measures $\mathcal{Q}$ such that

$$f(X) = \inf_{P \in \mathcal{Q}} \mathbf{E}_P X$$

Coherent risk measures G

Given an admissible set A and a norm in $\mathcal{X}$, generalized coherent risk measure is defined by

$$f(X) = (2I_A(X) - 1) \inf_{Y \in \partial A} \|X - Y\|, \quad X \in \mathcal{X}$$

Representation theorem G

f is a generalized coherent risk measure if and only if there exists a set $A_1^* \subseteq \mathcal{X}^*$ of linear functionals with unit norm such that

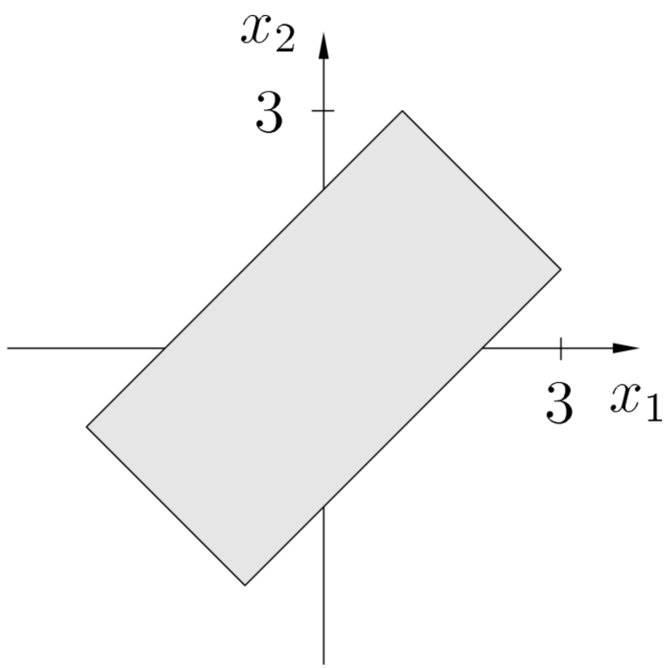
$$f(X) = \inf_{g \in A_1^*} g(X)$$

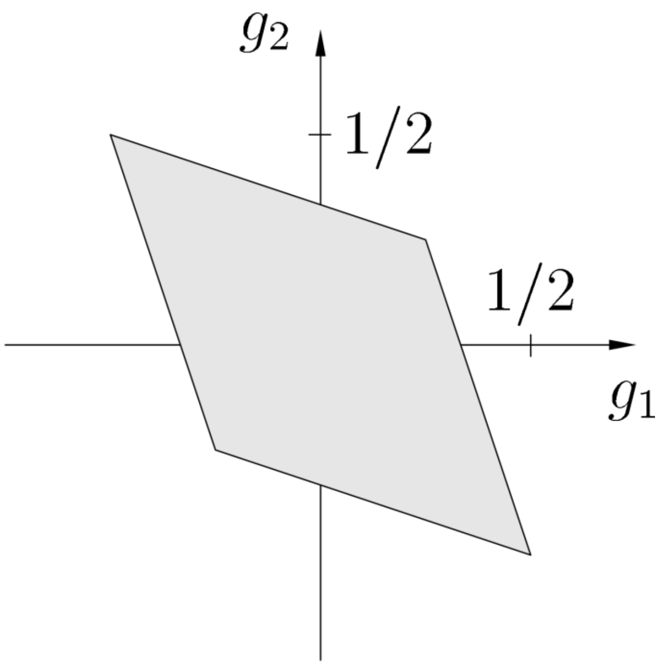
Norms for GCRM

L_p norms $\|X\|_p = \mathbf{E}|X|^p$

Minkowski functional $\|X\|_S = \inf\{r > 0 : X \in rS\}$

Example of Minkowski dual sphere





Unit sphere in $\mathcal{X}$

Unit sphere in $\mathcal{X}^*$